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Factorial moment analyses in diffractive lepton-nucleon scattering

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It is pointed out that ''the colorless objects'' in diffractive lepton-nucleon scattering in the small-x_B region can be probed by measuring the scaled factorial moments of final-state-hadrons of the diffractively produced system and the dependence of their scaling behavior upon the diffractive kinematic variables. The Monte Carlo implementations of RAPGAP and JETSET are discussed as illustrative examples. The results of these model calculations show in particular that the inclusion of the contributions from the gamma gluon fusion processes can considerably enlarge the power of the scaled factorial moments. The possibility for probing the anomalous scaling behaviors of probability moments of the transverse energies in the DESY ep collider HERA calorimeter environment is also discussed. [S0556-2821(98)50703-5]

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Recent experiments at the DESY ep collider (HERA) [1] on deep-inelastic electron-proton (ep) scattering (DIS) in the low x_B kinematic range have clearly shown the existence of a distinct class of events. These events are characterized by the fact that there is no hadronic energy flow in a considerably large interval of pseudorapidity η adjacent to the proton beam direction. Our present understanding of DIS could be inadequate at low-x_B because additions to the leading order QCD-based partonic picture are likely to be substantial. A natural interpretation of these so called ''large rapidity gap'' events is based on the hypothesis that the deep-inelastic scattering process involves the interaction of the virtual boson probe with a colorless component of the proton. Hence there is no chromodynamic radiation in the final state immediately adjacent to the direction of the scattered proton or any proton remnant. What is this colorless component originating from the nucleon? The large rapidity gap events discovered in deep inelastic scattering at HERA [1] are usually interpreted in terms of the pomeron-exchanged model [2]. Although this seems to work reasonably well phenomenologically, there is yet no satisfactory understanding of the pomeron structure and its interactions mechanism. In this respect, it is helpful to probe the properties of such objects in the nontraditional aspects, in addition to the traditional measures such as rapidity gap distribution, structure function and the averaged cross section and so on [1]. In the present note we wish to point out that useful information about the exchanged colorless object in diffractive lepton-nucleon scattering process can be extracted by studying scaling behavior and fractality (intermittency) of the final state of the colorless component in proton excited by the virtual boson probe.

The manifestation of fractality (intermittency) in high-energy multiparticle production processes is the anomalous scaling [3,4]

$$F_q(\delta x)=F_q(\Delta x)\left(\frac{\Delta x}{\delta x}\right)^{\phi_q} \quad \text{as } M\to\infty, \delta x\to 0 \tag{1}$$

of q-order factorial moments (FM's) F_q, defined as

$$F_q(\delta x)=\frac{1}{M}\sum_{m=1}^{M}\frac{\langle n_m(n_m-1)\cdots(n_m-q+1)\rangle}{\langle n_m\rangle^q}, \tag{2}$$

where x is some phase space variable, e.g., (pseudo)rapidity, the scale $\delta x=\Delta x/M$ is the bin width for a M partition of the region Δx in consideration, n_m is the multiplicity in the mth bin. Since the factorial moments (FM's) can remove the statistic noise around the probability and associated directly with the scaled moments of probabilities [3], the scaling exponent ϕ_q in Eq. (1), called intermittency index, characterizing the strength of dynamical fluctuation, is connected [5] with the anomalous fractal dimension d_q of rank q of the spatial-temporal evolution of high-energy collisions, $d_q=\phi_q/(q-1)$. A possible cause leading to the power-law of FM's of final particles in high-energy collisions is that the emitting source of final hadrons is self-similar fractal [6]. Furthermore, DIS experiments and the empirical analyses show that the gluon-density in the nucleon in the low-x_B kinematical region is much higher than those for quarks or antiquarks, and it is increasing with decreasing x_B [7]. In this soft gluon system where the gluons interact with each other in complicated dynamic processes, it has been proposed [8] that the gluons may ''self-organize'' into the clusters in the dissipative gluon system by self-organized criticality (SOC) [9], and the colorless component in proton can be regarded as color singlet gluon clusters [10]. The spatial-temporal structure of the SOC-cluster [or Bak-Tang-Wiesenfeld (BTW) cluster] is self-similar fractal [9]. So it is feasible to study the fractal structure of the colorless component of the proton by measuring the anomalous scaling behaviors of factorial moments of the final state particles originating from the scattering of virtual photon and colorless object in the diffractive lepton-nucleon scattering. While waiting for data to perform this analysis, let us at first consider the following two phenomenological models as illustrative examples:

(A) If the colorless object (c_0^*) is a quark-antiquark pair (formed by interacting gluons) which exists in the time-interval when the virtual photon γ^* is absorbed by the object, we shall see the following: Especially when Q^2 is sufficiently large, the incoming γ^* (the transverse dimension is expected to be proportional to $1/Q^2$) will hit the quark (q) or the antiquark ($\overline{q}$) and make them fly apart symmetrically with respect to the center of mass the $\gamma^* q\overline{q}$ system—similar to the $q\overline{q}$ pair produced in e^+e^- collisions (with respect to the center of mass the $q\bar{q}$ system). That is, in this case, the final-state-hadrons of an event are fragmentation products of the quark and/or the antiquark, and hence they are expected to show characteristic features similar as those observed in the reaction $e^+e^-\rightarrow$hadrons at the same c.m. system (c.m.s.) energy. It is interesting to notice that the very recent inclusive measurements performed at HERA [11] in diffractive electron-proton scattering show the following: Not only the scaled longitudinal momentum (x_F) distribution and the energy flow distribution, but also the "seagull" plot for average transverse momentum $\langle p_\perp \rangle$ vs x_F is strikingly symmetric with respect to the center of mass of the virtual photon and the struck colorless object; and the general features of these distributions are very much the same as those observed in electron-positron collision processes. These facts strongly suggest that a more detailed comparison between these two collision processes would be useful.

For this purpose, we made use of the Monte Carlo (MC) program JETSET [13] to simulate the $\gamma^* q\bar{q}$ interaction process. We generated 50,000 MC events, and calculated the second normalized factorial moment in three-dimensional $(\eta, p_\perp, \phi)$ phase space at the given $q\bar{q}$-c.m.s energy $\sqrt{s}$, where the pseudorapidity η, transverse momentum $p_\perp$, and the azimuthal angle ϕ are defined with respect to the sphericity axis of the event. The usual cumulative variables X translated from $x=(\eta, p_\perp, \phi)$, i.e., $X(x)=\int_{x_{\rm min}}^{x}\rho(x)dx/\int_{x_{\rm min}}^{x_{\rm max}}\rho(x)dx$, were used to rule out the enhancement of F_q from a nonuniform inclusive spectrum $\rho(x)$ of the final hadrons [12]. The obtained results have been collected in Fig. 1(a) in double logarithmic F_2 vs M plots for different $\sqrt{s}$ (or M_X, which is the invariant mass of the $\gamma^* c_0^*$ system in a corresponding diffractive lepton-nucleon scattering event). It is clear that the higher the invariant mass of $\gamma^* c_0^*$ is, the larger the power of FM's, i.e., the stronger the dynamic fluctuation is. In the very low invariant mass ($\sqrt{s}=4.5$ GeV, say), the powers of FM's become less than 0, which can be referred to the constraint of the momentum conservation in

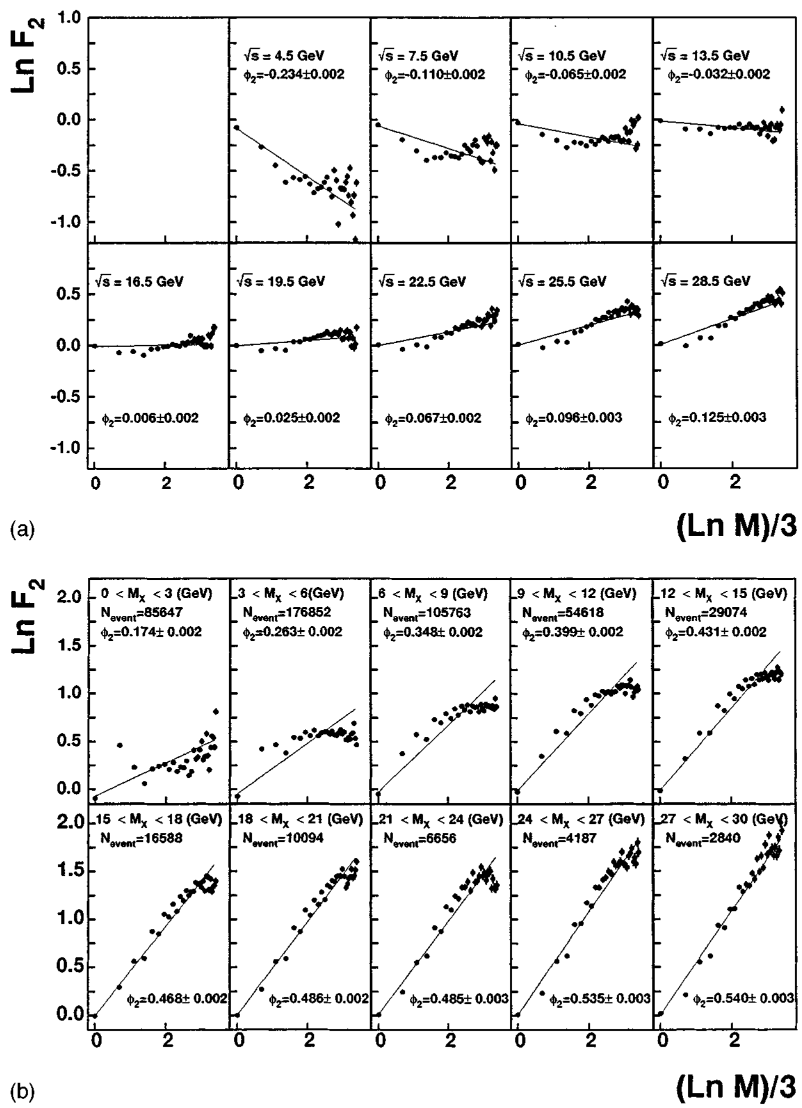

FIG. 1. The second-order scaled factorial moments F_2 versus the number M of subintervals of three-dimensional $(\eta, p_\perp, \phi)$ phase space in log-log plot, and the intermittency index ϕ_2 in corresponding sample set. (a) The MC result of JETSET 7.4 [13] in different c.m.s. energy $\sqrt{s}$, 50000 events are generated in each sample set; (b) that of RAPGAP [16] in corresponding invariable mass M_X interval with $N_{\rm event}$ events in each subsample.

RAPID COMMUNICATIONS

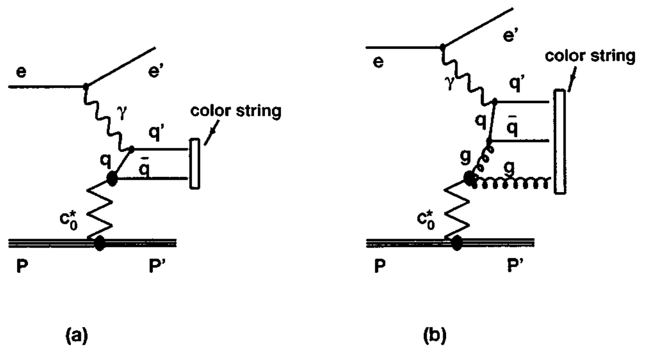

FIG. 2. The basic processes included in the RAPGAP [16] implementation for inelastic electron scattering on a pomeron: (a) the lowest-order process for hard parton level; (b) the $O(\alpha_{em}\alpha_s)$ order process for gamma gluon fusion.

the hadronization process [14].

(B) Based on pomeron-exchanged model [2], several Monte Carlo generators, such as POMPYT [15], RAPGAP [16] have been set up. These generators have been used to describe quite well HERA data in the global and averaged features, such as the rapidity gap distribution, the diffraction, and total cross section and so on [1]. The theoretic models are usually confronted with incorrigible discrepancy, when the data about the locally nonstatistic fluctuation in small phase space are involved in the comparison with them [4]. No evidence has shown that this kind of locally dynamic fluctuation could be certainly referred only to the hadronization process, but have nothing to do with the initial stage of high-energy collisions. In this respect, it is also relevant to see whether the features of scale invariance and fractality in small phase space of the lepton-nucleon diffractive processes, specially their dependence upon the diffractive variables can be compatible with the pomeron type of model. In the following we take RAPGAP as an example, in which the virtual photon (γ^*) interacts directly with a parton constituent of the pomeron either in lowest order [Fig. 2(a)] or in $O(\alpha_{em}\alpha_s)$ order—via the photon-gluon fusion [Fig. 2(b)]. In both cases a color octet remnant is left at low $p_\perp$ with respect to the pomeron and hence also to the proton. The higher-order gluon emission is simulated with the color dipole model (ARIADNE) [17] and the hadronization is performed using the JETSET [13]. The pomeron flux factor $f_{P\mathbb{P}}(t,x_\mathbb{P})$ and pomeron structure function $G(\beta)$ are taken, respectively, as

$$f_{P\mathbb{P}}(t,x_\mathbb{P})=\frac{\beta^2_{P\mathbb{P}}(t)}{16\pi}x_\mathbb{P}^{1-2\alpha_\mathbb{P}(t)}, \tag{3}$$

given by Berger *et al.* and Streng [2], and

$$\beta G(\beta)=6\beta(1-\beta), \tag{4}$$

when the pomeron is made of 2 ''unrealistically hard'' gluons as suggested by Ingelman and Schlein [2]. Here the kinematic variables t, $x_\mathbb{P}$, and β are defined as $t=(P-P')^2$, $x_\mathbb{P}=q\times(P-P')/(q\times P)$, and $\beta=-q^2/[2q\times(P-P')]$, where P, P', and q are the 4-momenta of incident nucleon, final state colorless remnant of nucleon, and virtual photon, respectively. The pomeron Regge trajectory is given by $\alpha_\mathbb{P}(t)=1+\epsilon+\alpha' t$, $\epsilon\approx 0.085$, and the slope $\alpha'=0.25$ obtained from a fit to data [2].

We generated 500,000 RAPGAP events, and divided the whole sample into 10 subsamples according to the invariable mass M_X of $\gamma^* c_0^*$ in the MC events. The scaling behaviors of FM's for different M_X intervals are shown in Fig. 1(b). The dependence of scaling behaviors of FM's upon M_X is similar with the result of JETSET in Fig. 1(a), i.e., the powers increase for increasing scattering energy of $\gamma^* c_0^*$. But it is noticeable that the intermittency index ϕ_2 for a given M_X interval is much larger in RAPGAP than that in JETSET. Having in mind that in Fig. 1(a) the colorless object is considered as a quark-antiquark pair formed by gluons and the Feymann graph of the diffractive process in this aspect is just same as shown in Fig. 2(a), the difference between cases (A) and (B) is that the higher-order photon-parton interaction has been taken into account in RAPGAP [see Fig. 2(b)]. It is understandable since the branch number of the parton cascading process in parton shower level is larger when the gamma gluon fusion is involved in Fig. 2(b), while it is believed generally that [4] power-law behaviors in the color-string type of models are referred in large part to the randomly cascading process of parton energy in perturbative phase, so the fractality in the evolution processes with longer cascade branch would be stronger.

Plenty of the experimental data [7] has observed a substantial increase in the gluon momentum density as x_B decreases in the small-x_B region. An interesting question is whether and how the fractal structure of diffractively produced system is influenced correspondingly by the increasing gluon density in small-x_B region. A simple and effective way for this is measuring the x_B dependence of scaling behaviors of FM's of the final-state-hadrons in diffractive process, and in particular that of intermittency index ϕ_q. In order to do so, we divided the RAPGAP MC sample into 10 subsamples according to x_B in the range of $0.05>x_B>0.0001$ in correspondence with HERA measurement region, and calculate the scaling index ϕ_2 of FM's for each subsample. The results are shown in Figs. 3(a), where a strong correlation between the intermittency index ϕ_2 and x_B is observed in this example. If c_0^* can be indeed considered as a pomeron as simulated in RAPGAP, the dependence of ϕ_2 on x_B shows that the smaller x_B is, the larger the gluon density is, the weaker the fractality of the spatial-temporal evolution processes originating from the $\gamma^* c_0^*$'s interaction will be.

An advantage of HERA data is the possibility to observe the small-x_B behavior over a large range of Q^2, which enable us to observe the dependence of fractal behavior of the diffractively produced system upon the space dimension ($\sim 1/Q^2$) of the virtual photon. In the Fig. 3(b), we present the Q^2 dependence of the intermittency index ϕ_2. Let us recall that, in photon-nucleon scattering experiments, not only those with real ($Q^2=0$) photons, but also those with spacelike ($Q^2>0$) photons where Q^2 is not too large ($\leqslant 1$ GeV$^2/c^2$, say) have very much in common with hadron-hadron collisions. It is well known that the index of intermittency for hadron-hadron scattering is smaller than that for electron-positron and lepton-nucleon scattering processes [4]. So the result in Fig. 3(b) shows that, the larger

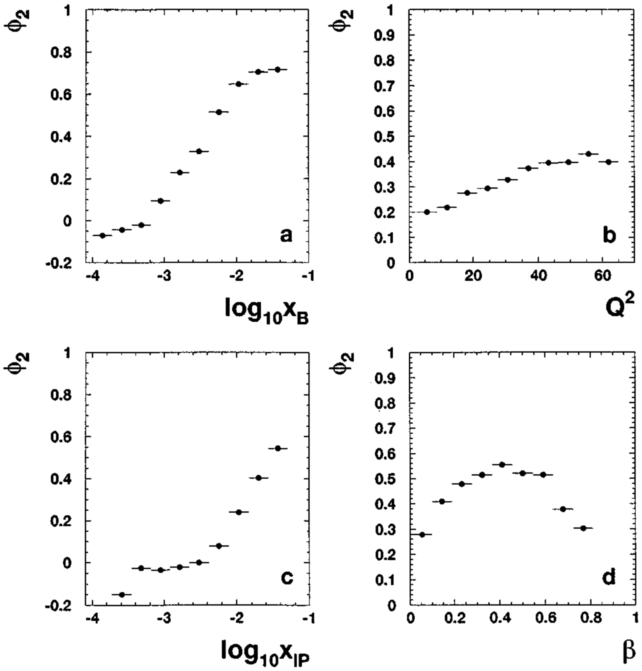

FIG. 3. The dependence of second-order intermittency index ϕ_2 in RAPGAP [16] MC implementation upon diffractive variables: (a) x_B, (b) Q^2, (c) x_{P}, and (d) β.

transverse dimension of virtual γ^* is, the more γ^* ''behaves like a hadron.'' In Figs. 3(c) and 3(d), we also present the dependence of scaling behavior upon x_{P} and β. It seems from RAPGAP MC generator that only when the momentum ($x_{\mathrm{P}}P$) of colorless object is large enough, can scaling behavior of the final state originating from $\gamma^* c_0^*$ be significant; and it is also clear from Fig. 3(d) that the dynamic fluctuation in the diffractive scattering does not increase monotonously for increasing the fraction β of momenta of stricken parton in the pomeron.

Last but not least, the following should be mentioned. Having in mind that jets have been observed [18] in diffractive electron-proton scattering processes, and HERA calorimeters have been used to measure transverse energies distribution of the collision events to study jet structure, it is meaningful to measure the scaling behavior of the probability moments of transverse energies in HERA calorimeters environment, instead of conventional multiplicity analysis. As is known [3], the factorial moments of multiplicity of final state particles, defined as Eq. (2), can rule out the statistical fluctuation around probability p_m by which a particles appear in the mth bin of phase space, i.e., $F_q = C_q \equiv (1/M)\Sigma_{m=1}^{M}\langle p_m^q\rangle/\langle p_m\rangle^q$. In order to measure the scaling behavior of probability moments of transverse energies, a straightforward manner following the usual procedure is to introduce an energy unit ε and write the ''transverse energy factorial moment'' $F_q^{(E)}$ as

$$F_q^{(E)} \equiv \frac{1}{M}\sum_{m=1}^{M}\frac{\langle E_{\perp m}(E_{\perp m}-\varepsilon)\cdots[E_{\perp m}-(q-1)\varepsilon]\rangle}{\langle E_{\perp m}\rangle^q}. \tag{5}$$

It is clear that $E_{\perp m}/\varepsilon$ can be considered as integers, provided that ε is sufficiently small. Under this condition, the statistical fluctuations around transverse energies can be removed in $F_q^{(E)}$ in the same way as that in F_q defined in Eq. (2). But, this means, there is a dependence on an arbitrary parameter ε, when we use $F_q^{(E)}$! In order to check the possibility of getting rid of this kind of arbitrariness in the practice, let us introduce a variable, $\lambda \equiv \varepsilon/E_\perp^t$, i.e., the ratio between the arbitrarily chosen energy unit ε and total transverse energy $E_\perp^t$ of an event. We generate at first the transverse energies of the ''events'' in the phase space by computer according to the Bernoulli distribution of λ. It is obvious that the slope in the double logarithmic $F_q^{(E)}$ vs M plot should be flat, since there is no dynamical fluctuation inputted in this sample. In this sample, we calculated the transverse energy moment

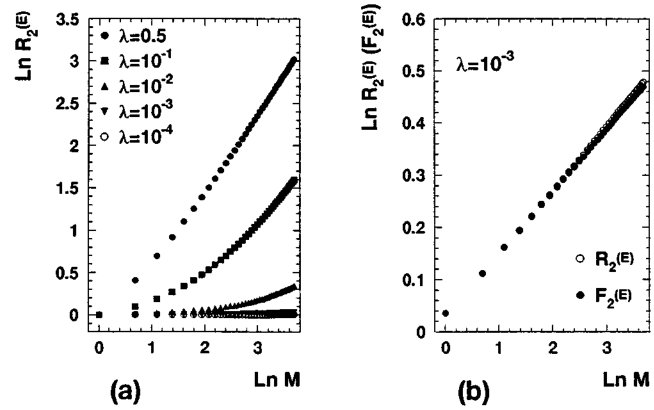

FIG. 4. (a) The second-order transverse energy scaled moment $R_2^{(E)}$ as functions of partition number of phase space $M = \Delta/\delta$ in log-log plot, when the transverse energy $E_\perp$ in units of ε in subinterval δ is stochastically produced according to Bernoulli distribution; (b) The comparison of $R_2^{(E)}$ and $F_2^{(E)}$, when the transverse energy $E_\perp$ in units of ε is produced according to the ''continued-scale'' randomly cascading α-model [19]. Here, $\lambda = \varepsilon/E_\perp^t$, and $E_\perp^t$ is the total transverse energy in the considered phase space Δ.

$$R_q^{(E)} \equiv \frac{1}{M}\sum_{m=1}^{M}\frac{\langle E_{\perp m}^q\rangle}{\langle E_{\perp m}\rangle^q}. \tag{6}$$

In Fig. 4(a) is the result of $R_2^{(E)}$ for different λ, which shows that the slope of $R_2^{(E)}$ vs M plot is also flat for small enough λ. Secondly, we let the transverse energies in unit of ε be produced according to the randomly cascading α-model with ''continued-scale'' [19] and compare the results of $R_2^{(E)}$ and $F_2^{(E)}$ when $\lambda = 10^{-3}$ [Fig. 4(b)]. In both cases, i.e., with and/or without nonstatistic fluctuations in the process of transverse energy productions, we see that the transverse energy moment $R_q^{(E)}$ can be considered as a good approximation for $F_q^{(E)}$, i.e., probability moments of transverse energies, when ε, which depends upon the resolving power of the calorimeters, is of the order of 10^{-3} of the total $E_\perp^t$ in the events under consideration.

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