

LETTER TO THE EDITOR

Characteristic plot of pomeron-exchange processes in diffractive DIS

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Abstract. The fractal behaviour of the pomeron-induced system in deep inelastic lepton–nucleon scattering is studied by Monte Carlo simulation of the RAPGAP generator. It is found that the dependence of the fractal exponent on the diffractive kinematic variables is rather robust and insensitive to the specific parametrization of the pomeron flux factor and structure function. This characteristic fractal plot can be considered as a clear experimental test of the pomeron model at the DESY ep collider HERA.

High-energy diffractive processes are usually described using the phenomenology of Regge theory [1] by the t -channel exchange of mesons and, at high energy, by the leading vacuum singularity, i.e. the pomeron [2].

Due to a lack of understanding of the nature of the pomeron and its reaction mechanisms, there exist different approaches to, and parametrizations of pomeron dynamics in current Regge theory, especially regarding the uncertainties of the hardness/softness of the gluon distribution inside the pomeron and the pomeron flux inside the hadron [3–6]. In recent years many theoretical calculations concerning the collective aspects of the diffractive processes, such as the cross section of hard diffraction [3–5], distribution of large rapidity gap [7, 8], jet production [5, 9, 10] of hard diffractive processes etc, have been performed.

Although much important information and insight into the diffractive process has been obtained, the results of the calculations were found to be very sensitive to the specific parametrization of the pomeron dynamics. So the experimental data concerning the collective features can be easily confounded by the phenomenological pomeron model when the unknown parametrizations are quite different in different data[†], which makes it rather difficult to conclude whether pomeron theory *indeed* works for the diffractive process. In this respect, it is natural to ask the following questions. Is there a way to test and justify the pomeron exchange model by using current diffractive experimental equipment (e.g., the DESY ep collider HERA) if the model is not based upon a specific parametrization of pomeron? If the answer is yes, then what is the characteristic behaviour of the pomeron exchange model expected in the experimental measurements?

[†] For example, the earlier data of the UA8 Collaboration [9] about jet production in CERN Sp̄pS-Collider indicated a soft gluon content of pomeron, while newer and more comprehensive data of the UA8 Collaboration [9] required a hard pomeron content; the analysis of structure function by ZEUS Collaboration [11] in DESY ep collider HERA needed both hard and soft components of the pomeron; the recent study of the structure function of QCD evolution by the H1 Collaboration [12] favoured a rather peculiar one-hard-gluon pomeron.

Bearing in mind that the fractal and fluctuation pattern of the multiparticle production reveals the nature of the correlations of the spatial-temporal evolutions in both levels of parton and hadron and is, therefore, sensitive to the interaction dynamics of the high-energy process [13]†, it is of interest to investigate the fractal behaviour of the diffractively produced system by calculating the scaled factorial moments of the multihadronic final state. In this letter, we find that the dependence of the fractal behaviour of the pomeron-induced system upon the diffractive kinematic variables is rather robust and insensitive to the different parametrization of the phenomenological pomeron model in the deep inelastic lepton–nucleon scattering (DIS). So these fractal characteristics can be considered as a clear experimental test of the pomeron exchange model at the DESY ep collider HERA.

The fractal (or intermittency) behaviour of the diffractively produced system in DIS (and also in hadron–hadron collisions) can be extracted by measuring the q -order scaled factorial moments (FMs) of the final-state hadrons excluding the intact proton from the incident beam, which are defined by [13]

$$F_q(\delta x) = \frac{1}{M} \sum_{m=1}^M \frac{\langle n_m(n_m - 1) \dots (n_m - q + 1) \rangle}{\langle n_m \rangle^q} \quad (1)$$

where x is some phase space variable of the multihadronic final state, e.g. (pseudo-)rapidity, the scale of phase space $\delta x = \Delta x/M$ is the bin width for a M -partition of the region Δx in consideration, n_m is the multiplicity of diffractively produced hadrons in the m th bin and $\langle \dots \rangle$ denotes the vertical average for different events for a fixed scale δx . Since the factorial moments F_q can rule out the statistic noise around probability p_m for a particle to be produced in the small phase space of the final state, and be associated directly with the scaled probability moments $C_q = \frac{1}{M} \sum_{m=1}^M \frac{\langle p_m^q \rangle}{\langle p_m \rangle^q}$, it is clear that F_q would saturate to some constant with decreasing δx to the some typical size δx_0 (e.g. the correlation length from resonance decays) if there were no local dynamic fluctuation in the multiparticle producing process and the probability distribution of the phase-space density was smooth, especially in the small scale of the phase space. The manifestation of the fractality and intermittency in high-energy multiparticle production refers to the anomalous scaling behaviour of FM [13, 14]

$$F_q(\delta x) \sim (\delta x)^{-\phi_q} \sim M^{\phi_q} \quad \text{as } M \rightarrow \infty, \quad \delta x \rightarrow 0. \quad (2)$$

The q -order intermittency index ϕ_q can be connected with the anomalous fractal dimension d_q of rank q of the spatial-temporal evolution of high-energy collisions [15], i.e. $d_q = \phi_q/(q - 1)$.

In the typical kinematic region for hard diffractive processes of the DESY ep collider HERA (say, $M_X > 1.1\text{GeV}$, and $x_{\mathbb{P}} < 0.1$), major properties of the diffractive events can be well reproduced by the RAPGAP generator (see, e.g. [7, 8, 10, 12]), based on the assumption that the diffractive process can be considered as taking place in two steps [3]. Firstly, a beam hadron emits a pomeron (\mathbb{P}) with longitudinal momentum fraction $x_{\mathbb{P}}$; in the second step this pomeron interacts with the other beam lepton in a large momentum transfer process between the basic constituents of the pomeron and the lepton. This pomeron factorization allows the diffractive hard scattering cross-section to be written as

$$\frac{d^4\sigma(\text{ep} \rightarrow \text{e} + \text{p} + \text{X})}{dx_{\mathbb{P}} dt d\beta dQ^2} = f_{p\mathbb{P}}(t, x_{\mathbb{P}}) \frac{d^2\sigma(\text{e}\mathbb{P} \rightarrow \text{e} + \text{X})}{d\beta dQ^2}, \quad (3)$$

where x_B and Q^2 are the usual deep-inelastic variables, and $\beta = x_B/x_{\mathbb{P}}$. The first factor on the RHS of equation (3) is the pomeron flux, i.e. probability of emitting a pomeron from the

† See e.g. [14] for a review of fractal (intermittency) in high-energy physics.

proton. The second factor, i.e. the lepton–pomeron hard cross section which is assumed to be independent of the negative mass-squared t of pomeron, could be calculated by

$$\frac{d^2\sigma(e\mathbb{P} \rightarrow e + X)}{d\beta dQ^2} = \int d\beta' G(\beta') \frac{d^2\hat{\sigma}_{\text{hard}}(e + \text{parton} \rightarrow e + X)}{d\beta dQ^2}, \quad (4)$$

if we could figure out the density $G(\beta')$ of the quarks and gluons with fraction β' of the pomeron momentum.

In dealing with the hard interactions between the virtual photon and the parton constituent of the pomeron, the RAPGAP generator [8] includes a quark–parton model diagram ($\gamma^*q \rightarrow q$), photon–gluon fusion ($\gamma^*g \rightarrow q\bar{q}$) and QCD–Compton ($\gamma^*q \rightarrow qg$) processes, which are generated according to the $O(\alpha_{em})$ and $O(\alpha_{em}\alpha_s)$ matrix elements, respectively. Higher-order QCD corrections are provided by the colour dipole model as implemented in ARIADNE [16], and the hadronization is performed using JETSET [17]. The QED radiative processes are included via an interface to the program HERACLES [18].

By choosing the specific pomeron flux factor $f_{p\mathbb{P}}(t, x_{\mathbb{P}})$ and structure function $G(\beta)$ which are already extensively used in current literature [3–6], we generate 100 000 RAPGAP MC events and calculate the second-order factorial moments in three-dimensional (η, p_{\perp}, ϕ) phase space, where the pseudorapidity η , transverse momentum p_{\perp} and the azimuthal angle ϕ are defined with respect to the sphericity axes of the events. The cumulative variables [19] translated from $x = (\eta, p_{\perp}, \phi)$, i.e.

$$X(x) = \int_{x_{\min}}^x \rho(x) dx \Big/ \int_{x_{\min}}^{x_{\max}} \rho(x) dx \quad (5)$$

are used to rule out the enhancement of FMs from a non-uniform inclusive spectrum $\rho(x)$ of the final produced particles. The obtained result of second-order FM versus the decreasing scale of the phase space is shown in figure 1 (log–log). There obviously exists anomalous scaling behaviour in the pomeron-induced interaction; we fit the points in figure 1 to equation (2) by the least-squares method.

The obtained intermittency indices ϕ_2 are shown in the corresponding plots.

In figures 1(a)–(c), we keep the pomeron structure function $G(\beta)$ unchanged but vary the pomeron flux $f_{p\mathbb{P}}(t, x_{\mathbb{P}})$. The fractal behaviour of the diffractively produced particles remains almost unchanged for different flux factors. Then, we keep the pomeron flux fixed in figures 1(a), (d)–(f), but vary the pomeron structure function. For a given pomeron flux, the fractal becomes weaker when the pomeron becomes softer. In figure 1(d) the parton distribution is as soft as that in a proton and the intermittency index is the smallest.

This is understandable since, when a harder parton in the pomeron is involved it is more possible to evoke jets and then the anomalous short-range correlation in the final state so that the intermittency index increases, and vice versa.

In order to see more closely the ability of intermittency measurements to attack pomeron dynamics, we also investigate the dependence of the fractal behaviour of the pomeron-induced system upon the diffractive kinematic variables as defined in equations (3) and (4). We generate 500 000 events by RAPGAP Monte Carlo generator, and divide the whole event-sample into ten subsamples according to the diffractive kinematic variables, e.g. x_B . For each subsample, we calculate the second-order scaled FM and the intermittency index ϕ_2 . Some interesting features can be obtained from the dependence of the second-order intermittency index ϕ_2 on the different diffractive kinematic variables:

- (1) It has been shown from high-energy hadron–hadron, hadron–nucleus and nucleus–nucleus collisions [14, 15] that when a fractal system consists of several fractal sources, the

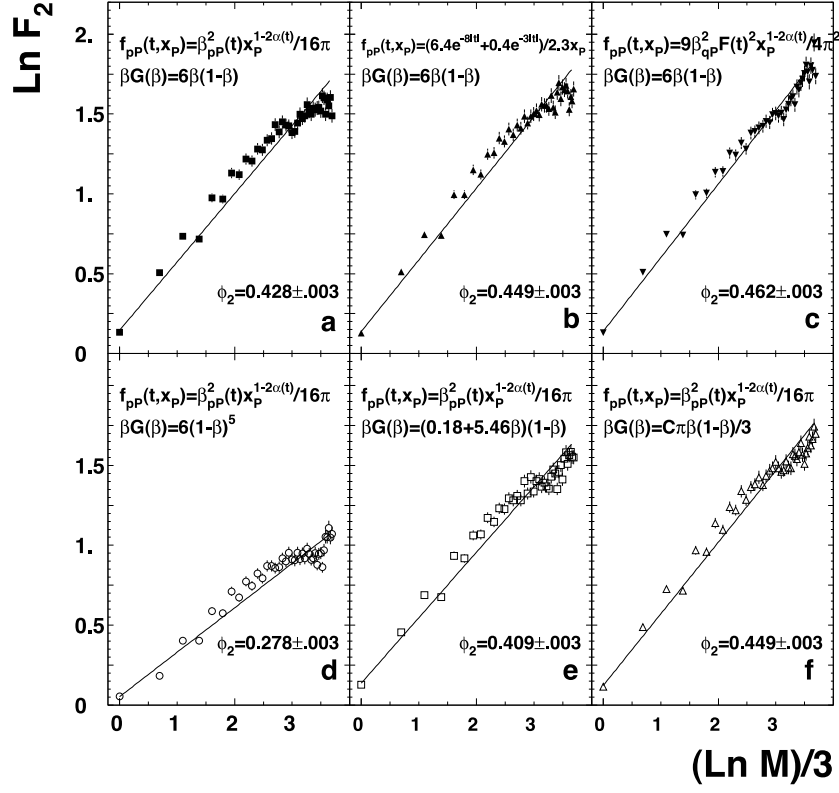


Figure 1. The second-order scaled factorial moments F_2 obtained from MC simulation of the RAPGAP generator [8] versus the number M of subintervals of three-dimensional (η, p_{\perp}, ϕ) phase space in log-log plot, and the intermimty index ϕ_2 correspondingly. Different parametrization of the pomeron flux factor $f_{pp}(t, x_p)$ and structure function $G(\beta)$ are used in different plots (for details of the chosen parametrizations, see [3–6]).

superposition effect of the fractal sources will significantly weaken the intermimty of the system.

Moreover, the intermimty strength of the whole system decreases when the number of fractal sources increases.

This effect also exists in the diffractive deep inelastic process of figure 2(a). Since it is well known that the gluon density increases sharply as x_B decreases in the small- x_B region [20], the dependence of ϕ_2 on x_B in figure 2(a) means that the anomalous fractal dimension d_2 of the diffractively produced system decreases with increasing gluon density. In other words, the gluon sources with higher density in the pomeron induce the lower intermimty in the final hadronic states.

- (2) Since the intermimty calculation concerns the multiplicity measurement in the final hadronic level after a diffractive deep inelastic event, the interesting question arises, of whether this fractal analysis indeed reflects not only the hadronization of the diffractively produced system but also, more importantly, the initial stage of the pomeron exchange process. The obvious dependence of ϕ_2 on the pomeron momentum fraction x_{pP} of a hadron in figure 2(b) and on the parton momentum fraction β of a pomeron in figure 2(c) (and also the dependence of FM on the softness/hardness of gluon distribution as shown

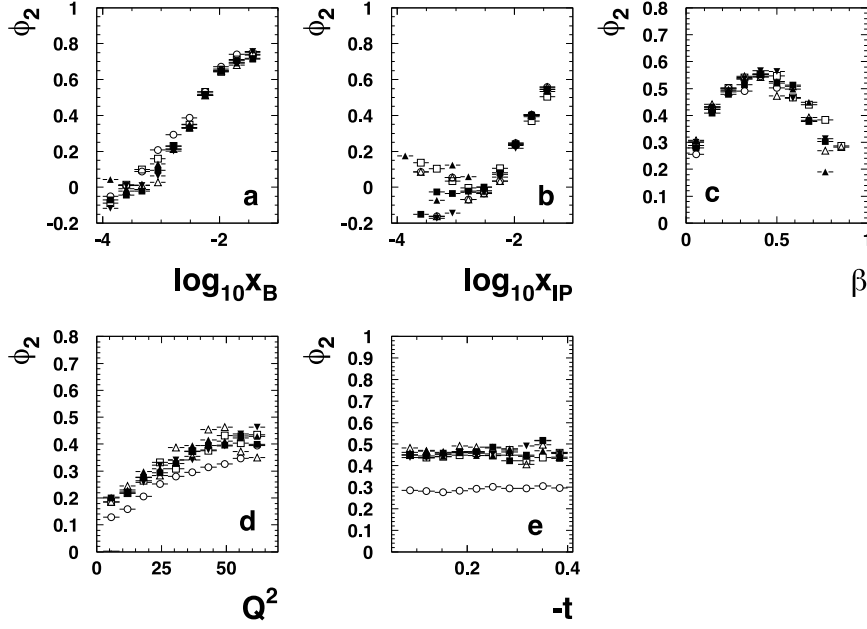


Figure 2. The second-order intermittency index ϕ_2 in the RAPGAP [8] Monte Carlo implementation versus different kinematic variables. The different shapes of points denote different parametrizations of pomeron flux factors $f_{p\mathbb{P}}(t, x_{\mathbb{P}})$ and the structure functions $G(\beta)$, as in figure 1.

in figure 1) show that the answer to the question is affirmative.

While the fractality of the diffractively produced system increases with increasing pomeron momentum, its dependence upon partonic momentum is not monotonous. It is noticeable that in figures 2(a) and (b) the intermittency index ϕ_2 is less than zero for lower x_B and $x_{\mathbb{P}}$. This effect is due to the constraint of the transverse momentum conservation in the high-energy process and can be partly eliminated by the method of ‘quadrant analysis’ [21].

- (3) An advantage of HERA data is the possibility of studying the small- x_B behaviour over a large range of Q^2 , which enables us to observe the dependence of the fractal behaviour of the diffractively produced system upon the space dimension ($\sim \frac{1}{Q^2}$) of the virtual photon. In figure 2(d), we present the Q^2 -dependence of the intermittency index ϕ_2 . Let us recall that, in photon–hadron scattering experiments, not only those with real ($Q^2 = 0$) photons but also those with space-like ($Q^2 > 0$) photons where Q^2 is not too large ($\leq 1\text{GeV}^2/c^2$, say) have a great deal in common with hadron–hadron collisions. It is well known that the index of intermittency of hadron–hadron scattering is smaller than that of electron–positron and lepton–nucleon scattering processes [14]. So the result in figure 2(d) shows that, the larger transverse dimension of virtual γ^* is, the more γ^* ‘behaves like a hadron’.
- (4) Since the leading proton spectrometer (LPS) has been used in the ZEUS detector to detect protons scattered at very small angles (say, ≤ 1 mrad), which make it possible to measure precisely the square of the four-momentum transfer t at the proton vertex, we also show, in figure 2(e), the t -dependence of the second-order intermittency index in the t -region of the LPS detector, i.e. $0.07 < -t < 0.4 \text{ GeV}^2$. Unlike the results of other kinematic variables, the fractal index for the $\gamma^*\mathbb{P}$ system does not depend upon t .
- (5) As mentioned above, the pomeron theory has been compared with the data of cross section of hard diffraction [3–5], the rapidity distribution of large rapidity gap [7, 8], and

jet productions [5, 9, 10, 10] etc. Although the conventional investigations are extremely relevant and plenty of important information and insight has been obtained (see references above), the calculations were found to be very sensitive to the uncertainties of the pomeron, and the experimental data in different aspects preferred different kinds of parametrization [9–12]. So it is rather inconclusive as to the feasibility of the pomeron model to describe the diffractive process if only relying on the investigations concerning the collective natures of the diffractive process. On the other hand, the intermittency calculation detects the local dynamical fluctuation in the small scale of the phase space and reflects the *inherent* scaling behaviour of the diffractive process [13, 14]. It is reasonable to expect that intermittency measurements can be robust and insensitive to the phenomenological uncertainties of the pomeron models while being more dependent on the spatial-temporal evolution structure of the pomeron dynamics. In order to show this point more clearly, we collect all the intermittency indices calculated from different kinds of pomeron flux and structure function in figure 2. The different shapes of points here denote different kinds of pomeron parametrizations, as in figure 1. The dependence of the intermittency index upon the diffractive kinematic variables are indeed rather robust and almost the same for the different parametrizations! Thus the typical fractal behaviour of the diffractively produced multihadron system shown in figure 2 can be considered as a characteristic plot of pomeron-exchange processes.

Bearing in mind that the multiplicity measurement is available at the DESY ep collider HERA, and especially that the multiplicity moments and KNO scaling behaviours of final hadrons in deep-inelastic processes have already been studied in recent years [22], it is feasible and urgent to check this characteristic fractal plot at the DESY ep collider HERA. However, it should be mentioned that the number of MC events in our calculations is up to 500 000, and the statistic error for the intermittency is rather small (the error bar in figure 2 is inside the points and not shown). In diffractive DIS experiments at the DESY ep collider HERA, the number of events selected for the multihadronic analysis are usually much smaller (e.g., ZEUS and H1 Collaborations have used 2748 and 4738 events, respectively [22]) and experimental error can be much larger than the calculations presented here. Nevertheless, the main tendency and dependence of intermittency indices on the diffractive kinematic variables should be comparable.

So substantial revision would be necessary in the manner in which we have treated diffraction if it turned out that experimental intermittency measurements differed *drastically* from the characteristic plot presented here.

Last but not least, the following should be mentioned. In figure 2 we only considered the uncertainties of the pomeron model in the aspects of hardness/softness of gluon distribution and pomeron flux. There also exist, however, uncertainties in the Q^2 evolution of parton densities of the pomeron. Numerical calculations using a QCD evolution equation have shown that [23] the Q^2 evolution of the pomeron structure function can be very different according to whether or not the nonlinear recombination term of the QCD evolution equation is included and which kind of initial parton distributions at a given momentum scale are chosen. By assuming both leading and subleading Regge trajectory, a fit of the QCD evolution equations to HERA data of $F_2^{D(3)}(x_{\mathbb{P}}, \beta, Q^2)$ has favoured a rather peculiar ‘one-hard-gluon’ distribution for the pomeron [12] (see also footnote†). Since what we try to pursue in figure 2 is to find out whether and in what range the fractal behaviour of the pomeron-induced system depends on varieties of different parametrization of pomeron, we leave the anomalous scaling behaviour in QCD evolution processes and possible influence from the pomeron uncertainties to further discussions [24].

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